

# INTERFEROMETRIC MEASUREMENT OF INSTANTANEOUS LOCAL HEAT TRANSFER FROM A HORIZONTALLY VIBRATING ISOTHERMAL CYLINDER\*

W. W. CARR

Product Technology, Monsanto Corporation, Pensacola, Florida 32500, U.S.A.

and

W. Z. BLACK

School of Mechanical Engineering, Georgia Institute of Technology, Atlanta, Georgia 30332, U.S.A.

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**Abstract**—Local instantaneous heat-transfer coefficients were measured for free convection from an isothermal cylinder vibrating sinusoidally in a horizontal plane. The results show that the heat-transfer coefficient is strongly dependent upon the ratio of amplitude of vibration to cylinder diameter,  $A/D$ . For values of  $A/D$  less than approximately 0.25, a critical vibration intensity was observed below which vibration caused only small increases in the heat-transfer rate; however, at  $A/D$  greater than 0.25, the critical vibrational intensity did not appear to exist because the heat transfer from the cylinder increased as soon as the vibrational speed increased from zero. A correlation of the data is presented which predicts the increase in average heat transfer from the cylinder to air above the free convective rate.

The correlation also applies to data for much larger vibrational intensities.

## NOMENCLATURE

- $A$ , amplitude of oscillation from centerline to extreme of oscillation;  
 $D$ , cylinder diameter;  
 $f$ , frequency of oscillation [c/s];  
 $Gr$ , Grashof number;  
 $Nu$ , Nusselt number;  
 $Nu_w$ , vibrational Nusselt number;  
 $\overline{Nu}_F$ , free convective Nusselt number;  
 $\overline{Nu}_{osc}$ , oscillatory Nusselt number;  
 $\overline{Nu}_w$ , overall average vibrational Nusselt number;  
 $Pr$ , Prandtl number;  
 $Re_A$ , average Reynolds number;  
 $\Theta$ , angular position around cylindrical surface ( $\Theta = 0$  at geometrical bottom);  
 $\Phi$ , augmentation factor — percent increase in free convective Nusselt number due to vibration;  
 $\Psi$ , angular position which locates the cylinder throughout cycle.

## INTRODUCTION

THE NEED for more efficient thermal systems stemming from industrial and space program applications has stimulated an interest in methods to augment heat transfer. One method of augmentation that has been investigated has involved the use of vibration. Both

mechanical vibrations where the surface is moved relative to the stationary surrounding fluid and acoustical vibrations where the fluid is moved relative to a stationary cylinder have been used in studies involving both free and forced convection. A frequently used geometry for these investigations has been a horizontal cylinder vibrating in either a horizontal or vertical plane.

Over the last thirty years, there have been several investigations of the effect of vibrations on the average heat-transfer rate from wires and cylinders in free convection [1–10]. A parameter that has proved to be important in the oscillating cylinder problem is the ratio of the amplitude of oscillation to cylinder diameter,  $A/D$ . This vibrational parameter indicates the type of process in the vicinity of the cylinder. For  $A/D \gg 1$ , a quasi-steady type flow exists. Three regions of flow are found: (1) a region where free convection dominates; (2) a transitional region of both free and forced convection; and (3) a region where forced convection dominates.

For  $A/D \ll 1$ , there is no periodic displacement of fluid across the cylinder to effect a net increase in enthalpy transport. In this case, natural convection dominates until the critical vibrational intensity is reached. Then, a type of coupling between streaming currents due to vibration and natural convective currents causes a net increase in enthalpy transport.

\*Based, in part, on the doctoral dissertation of Carr [16].

This phenomenon is called thermoacoustic streaming. For intermediate values of  $A/D$ , the flow pattern is more complex. All three ranges of this parameter (small  $A/D$  [1–4], medium  $A/D$  [5–7], and large  $A/D$  [8–10]) have been investigated.

There have been many investigations of the effect of mechanical vibration on the average heat-transfer rate (averaged over both time and the surface of the cylinder) from an isothermal, horizontal cylinder to air in free convection, but no measurements have been made of local heat-transfer coefficients. However, two groups [11–15] of investigators have reported on the effect of acoustical vibration on the average local heat-transfer rate from an isothermal, horizontal cylinder to air in free convection. The terminology average local heat-transfer coefficient is used here to specify a heat-transfer coefficient at one position on the surface of the cylinder that has been averaged over a cycle of oscillation. Since these investigations were made for acoustical vibrations, they are limited to small ratios of amplitude to diameter which are approximately two orders of magnitude smaller than those reported in this study.

No measurement of instantaneous local heat-transfer coefficients have been reported for either acoustic or mechanical vibration. The term instantaneous local heat-transfer coefficient refers to a heat-transfer coefficient at one point on the surface of the cylinder that was measured at one instant of time. The lack of instantaneous local data is probably due to the absence of good technique of measurement.

This paper is concerned with the effect of horizontal, transverse, mechanical vibrations on the heat-transfer rate from an isothermal cylinder to air in free convection. Instantaneous local heat-transfer data that were obtained using a differential interferometer are presented. In addition, instantaneous average, average local, and overall average heat-transfer results are discussed.

#### APPARATUS

A differential interferometer was used in this study to obtain instantaneous, local heat-transfer data. Other optical instruments such as the Mach–Zehnder interferometer have been used extensively for heat-transfer measurements; however, the differential interferometer possesses unique capabilities for measuring local heat-transfer coefficients under rapidly changing transient conditions. This advantage is due to the fact that the differential interferometer produces fringe shifts proportional to local temperature gradients. As a result, local heat-transfer coefficients can be readily measured with this device which makes it a particularly good choice for the measurement of the instantaneous local heat-transfer coefficient from a vibrating cylinder. A

discussion of the differential interferometer and its optical components may be found in [16–18].

A gold plated, copper cylinder having a diameter of 2.5 cm and a length of 30.3 cm was used for both the stationary and vibratory free convective tests. The cylinder was heated electrically by a tubular heating element located along the axis of the cylinder. Horizontal, sinusoidal motion of the test cylinder was produced by a Scotch yoke mechanism that utilized counterbalancing weights to eliminate the inertia forces of the yoke and cylinder. The fringe pattern produced by the temperature gradients surrounding the cylinder was photographed throughout the cycle of oscillation with a 16 mm high-speed camera. The fringe shifts at the surface of the cylinder were measured from the photographs and were used to evaluate the heat-transfer coefficients.

#### EXPERIMENTAL MEASUREMENTS

The experimental measurements reported here were divided into two phases. First, local heat-transfer coefficients for a stationary isothermal cylinder placed in air at one atmosphere pressure were measured. These stationary tests were repeated several times to establish reliable values since they were to be used as the basis of comparison with vibratory tests. The second phase consisted of the measurement of instantaneous local heat-transfer coefficients under vibratory conditions. A series of tests were conducted at the same Grashof numbers as those of the stationary tests but at various amplitude to cylinder diameter ratios,  $A/D$ , and average Reynolds numbers,  $Re_A$ .

The ranges of parameters investigated were:

$$2.44 \times 10^4 \leq Gr \leq 7.60 \times 10^4$$

$$0 \leq \frac{A}{D} \leq 1.78$$

$$0 \leq Re_A \leq 660.$$

The lower limit of the Grashof number corresponds to an excess temperature of 14 deg C while the upper limit value corresponds to an excess temperature of 56 deg C. The average Reynolds number has the average cylinder speed (four times the  $Af$  product) as a characteristic velocity and the cylinder diameter as the characteristic length. The upper limit on the Reynolds number corresponds to an  $Af$  product of 11 cm/s.

The instantaneous local Nusselt numbers  $Nu_V(\Theta, \Psi)$  were measured for seven angular positions  $\Psi$  throughout the cycle of oscillation and for twenty-four angular positions  $\Theta$  around the surface of the cylinder with the geometric bottom of the cylinder being denoted by  $\Theta = 0$ . At each position of  $\Psi$ , an instantaneous average Nusselt number,  $Nu_V(\Psi)$ , was calculated by averaging the instantaneous local values over the

surface of the cylinder (integrated over  $\Theta$ ). Similarly, an average local Nusselt number,  $Nu_{\nu}(\Theta)$ , was calculated at each position by averaging the instantaneous local values over an entire cycle of oscillation (integrated over  $\Psi$ ). An overall average Nusselt number,  $\bar{Nu}_{\nu}$ , was calculated by averaging the instantaneous average values over the entire cycle of oscillation.

For the analysis which follows, the Nusselt number is considered to be a function of four dimensionless parameters or

$$Nu = Nu\left(Pr, Re_A, Gr, \frac{A}{D}\right). \quad (1)$$

This functional relationship has been used previously in other studies involving heat transfer from vibrating cylinders. Lemlich [5] performed a dimensional analysis to show that the Nusselt number for a vibrating cylinder in free convection is a function of these four dimensionless groups, and Dougall *et al.* [19] have analyzed this problem by writing the governing equations and boundary conditions in dimensionless form and have recommended this set of dimensionless parameters as a promising combination.

RESULTS

Instantaneous, local heat-transfer measurements were made for all of the vibrational tests. Instantaneous local increases up to 1400 per cent and instantaneous local decreases as large as 80 per cent were measured. The results of several representative tests presented in Figs. 1-4 illustrate the effect of vibration on the instantaneous local Nusselt number,  $Nu_{\nu}(\Theta, \Psi)$ . Each figure consists of eight polar plots showing the variation of  $Nu_{\nu}(\Theta, \Psi)$  with position around the cylinder surface.

Seven of the plots correspond to seven different  $\Psi$  locations while the eighth plot shows the local Nusselt number for a stationary cylinder.

Figures 1-3 show the effect of varying  $A/D$  on  $Nu_{\nu}(\Theta, \Psi)$  for a Grashof number of approximately  $2.5 \times 10^4$  and an average Reynolds number of approximately 150. By comparing the seven vibratory plots with the single stationary plot in Fig. 1, it can be seen that the effect of vibration on  $Nu_{\nu}(\Theta, \Psi)$  was very small for  $A/D$  of 0.118. At a ratio of amplitude to diameter of 0.237, the effect due to vibration was also insignificant.

In Fig. 2 the effect of vibration at  $A/D$  of 0.501 is much more pronounced than that at lower amplitude to diameter ratios. The shapes of the  $Nu_{\nu}(\Theta, \Psi)$  polar plots resemble forced convective curves for all  $\Psi$  positions except 30 and 60°. At  $\Psi$  equal to 30°, the polar variation in the local Nusselt number strongly resembles the stationary free convective plot. At  $\Psi$  equal 60°, the plot has both free and forced convective characteristics with the shape of the curve losing some of its similarity to the stationary plot while gaining a resemblance to the forced convection curve. The Nusselt number on the leading half of the cylinder is becoming larger than  $Nu_{\nu}(\Theta, \Psi)$  on the trailing half.

Variations in the local Nusselt number throughout the cycle for  $A/D$  of 1.78 are shown in Fig. 3. The most pronounced differences between the plots for  $A/D$  of 0.501 and 1.78 occur at  $\Psi$  positions of 30 and 60°. For  $A/D$  of 1.78 and  $\Psi$  of 60°, the polar plot of  $Nu_{\nu}(\Theta, \Psi)$  resembles a forced convective plot and  $Nu_{\nu}(\Theta, \Psi)$  at  $\Psi$  of 30° has both free and forced convective characteristics similar to the  $Nu_{\nu}(\Theta, \Psi)$  plot for  $A/D$  of 0.501 at  $\Psi$  of 60°.

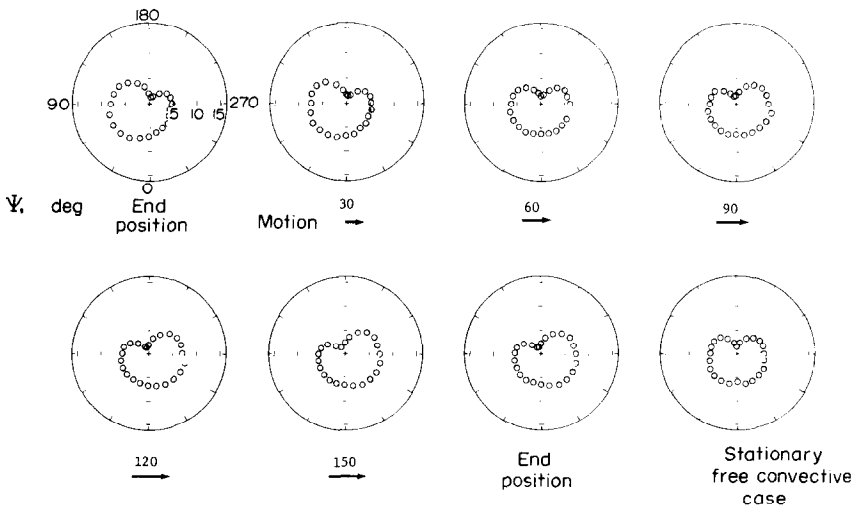


FIG. 1. Distribution of  $Nu_{\nu}(\Theta, \Psi)$  with  $\Theta$  for  $A/D = 0.118$ ,  $Re_A = 141$  and  $Gr = 2.5 \times 10^4$ .

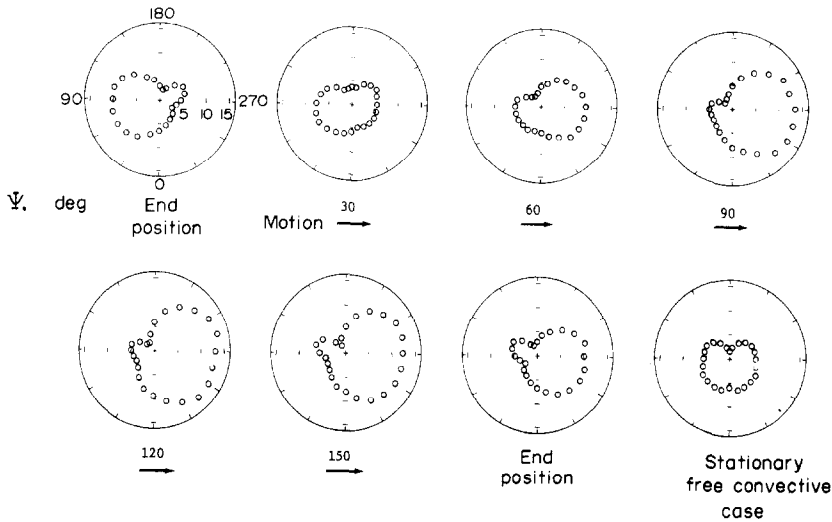


FIG. 2. Distribution of  $Nu_{\nu}(\Theta, \Psi)$  with  $\Theta$  for  $A/D = 0.501$ ,  $Re_A = 151$  and  $Gr = 2.6 \times 10^4$ .

The differences in the plots of  $Nu_{\nu}(\Psi)$  at  $\Psi = 30$  and  $60^\circ$  are due to the fact that the frequency of oscillation is higher for lower values of  $A/D$ , and the acceleration of the cylinder is higher. Thus, at the smaller  $A/D$  ratio, the cylinder changes direction more quickly at the extreme position of oscillation, and the air heated in the previous half cycle of oscillation has less time to rise out of the cylinder's path. Therefore, the heat transfer from the cylinder is more affected by the previous half cycle of oscillation.

Comparison of Figs. 2 and 4 shows the effect of increasing  $Re_A$  from approximately 150 to 600 at a Grashof number of approximately  $2.5 \times 10^4$  and  $A/D$

of approximately 0.5. The shapes of the curves for a given  $\Psi$  position are not drastically altered as  $Re_A$  is increased even through the magnitude of  $Nu_{\nu}(\Theta, \Psi)$  significantly increased.

Figure 4 shows that as the cylinder passes through the midpoint in an oscillation, the Nusselt number in the vicinity of the wake of the cylinder ( $\Theta = 90^\circ$ ) increases even though the velocity of the cylinder is decreasing. This phenomena is in contrast to the trend predicted by a pure forced flow model where there is a region of relatively low heat transfer at the rearward side of the cylinder. Observation of the infinite fringe motion pictures indicated that due to the deceleration

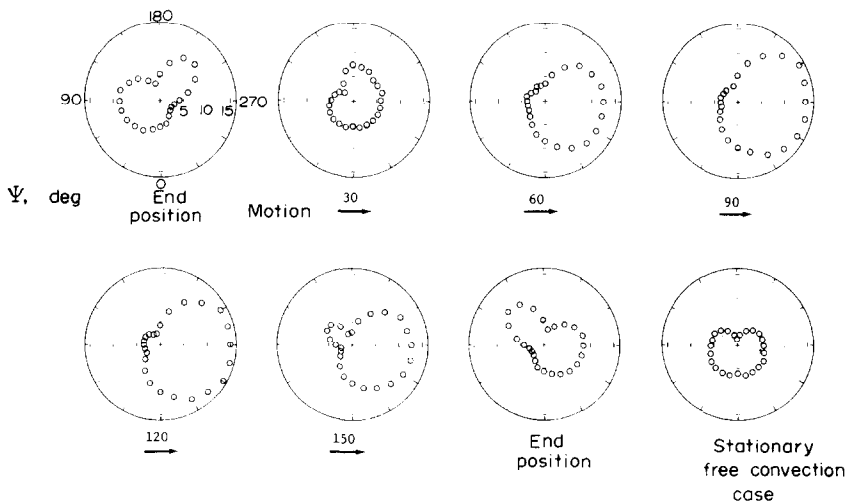


FIG. 3. Distribution of  $Nu_{\nu}(\Theta, \Psi)$  with  $\Theta$  for  $A/D = 1.78$ ,  $Re_A = 155$  and  $Gr = 2.6 \times 10^4$ .

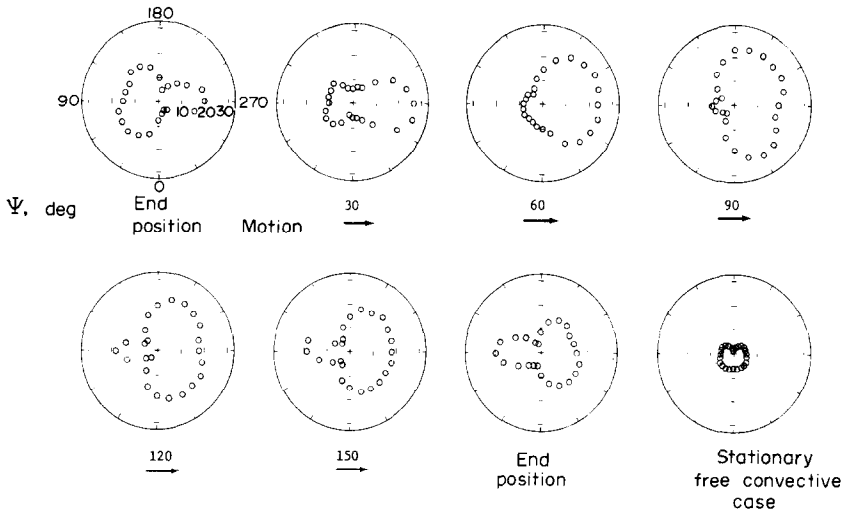


FIG. 4. Distribution of  $Nu_v(\Theta, \Psi)$  with  $\Theta$  for  $A/D = 0.518$ ,  $Re_A = 617$  and  $Gr = 2.5 \times 10^4$ .

of the cylinder at the higher speeds, the flow separations on the trailing side of the cylinder moved toward the cylinder causing vigorous circulation in the wake that increased the heat-transfer rate on the trailing side of the cylinder.

In general, increasing the Grashof number from  $2.5 \times 10^4$  to  $7.5 \times 10^4$  resulted in an increase in the magnitude of  $Nu_v(\Theta, \Psi)$  while the shapes of the  $Nu_v(\Theta, \Psi)$  plots changed very little. The increase in the magnitude of  $Nu_v(\Theta, \Psi)$  was much more pronounced near the extremes of oscillation where the buoyancy forces were strongest while the increase was

much smaller near the center of the cycle of oscillation where the inertial forces were most significant. The stronger effect of  $Gr$  near the extreme position can be observed in Fig. 2 where the shape of the curve very closely resembles the free convective curve at  $\Psi = 30^\circ$ , but at the center of the oscillation, the curve resembles the force convective curve.

Instantaneous average Nusselt numbers,  $Nu_v(\Psi)$ , were calculated for all of the vibratory tests by averaging the instantaneous local Nusselt numbers over the entire cylinder surface. The largest instantaneous increase measured was 240 per cent while the

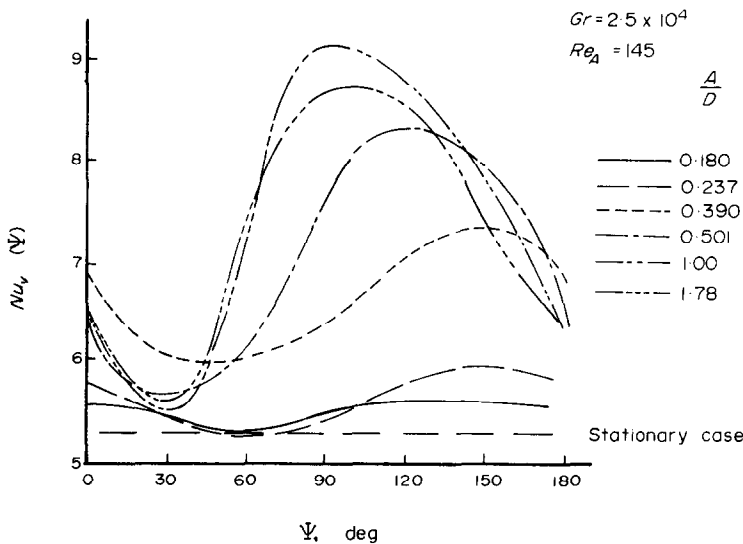


FIG. 5. Variation in instantaneous average Nusselt number,  $Nu_v(\Psi)$  with the amplitude to diameter ratio,  $A/D$ .

largest instantaneous decrease was 4 per cent. Plots of  $Nu_V(\Psi)$  showed distinct variations with  $A/D$ . The trends in these variations can be seen in Fig. 5. At low  $A/D$ , the minimum values of  $Nu_V(\Psi)$  were in the vicinity of  $\Psi = 60^\circ$ , but as  $A/D$  increased, the minimum moved toward lower values of  $\Psi$  until it was located between 0 and  $30^\circ$ . The  $\Psi$  position of the maximum value of  $Nu_V(\Psi)$  also varied with  $A/D$ . At low  $A/D$ , the  $\Psi$  position of the maximum was positioned between 150 and  $180^\circ$ , but as  $A/D$  increased the  $\Psi$  position of the maximum moved toward the center of oscillation until the maximum was near  $\Psi = 90^\circ$ .

These trends can be explained by the fact that at large  $A/D$  the horizontal motion of the cylinder carried the cylinder much beyond the free convective boundary layer into cooler air resulting in a free-forced type flow near the center of oscillation ( $\Psi = 90^\circ$ ). The decrease in  $Nu_V(\Psi)$  for  $\Psi$  values near  $30^\circ$  is due to the cylinder having moved back into air heated in the previous cycle. At small  $A/D$ , the horizontal motion of the cylinder was smaller than the free convective boundary-layer thickness, and the cylinder did not transverse outside this layer. The flow pattern near the center of oscillation closely resembled free convection.

In general, increasing  $Re_A$  resulted in an increase in  $Nu_V(\Psi)$  at all  $\Psi$  positions, and the largest increase in  $Nu_V(\Psi)$  occurred in the vicinity of  $\Psi$  equal to  $90^\circ$  where the cylinder velocity was near its maximum value. The value of  $Nu_V(\Psi)$  at  $\Psi = 0$  degrees increased with increasing  $Re_A$  even though the cylinder velocity was zero at this position. This increase in heat dissipation was due to the fact that there was relative motion between the air and the cylinder at the stroke of oscillation even though the velocity of the cylinder was zero. At high  $Re_A$ , this motion appeared to be fairly turbulent.

Increasing  $Gr$  caused an increase in  $Nu_V(\Psi)$ , but the effect of vibration on the Nusselt number was greatest at low  $Gr$  when the buoyancy forces were weakest. The effect of  $Gr$  was smallest near the center of oscillation where the inertia forces were strongest and largest near the extreme positions of oscillation where the inertia forces were weakest.

Average, local Nusselt numbers (local values averaged over a representative cycle),  $Nu_V(\Theta)$ , were calculated by averaging twelve instantaneous local Nusselt numbers over one cycle of oscillation. The distribution of  $Nu_V(\Theta)$  is plotted for two representative tests in Fig. 6. Case (a) is for  $A/D = 0.41$ ,  $Gr = 2.5 \times 10^4$ , and  $Re_A = 660$ . The dimensionless numbers correspond to a frequency of oscillation of  $10.7 \text{ c/s}$  and an excess temperature of  $14 \text{ deg C}$ . Comparison of  $Nu_V(\Theta)$  with the stationary values in Fig. 6 shows that vibration increased the heat transfer at all positions around the

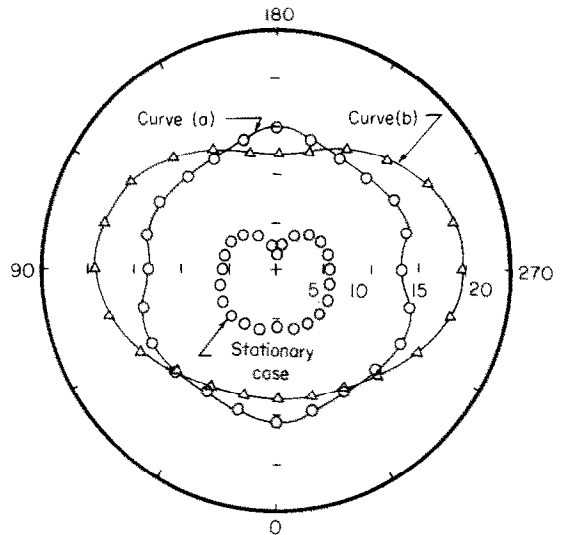


FIG. 6(a). Distribution of  $Nu_V(\Theta)$  with  $\Theta$  for  $A/D = 0.41$ ,  $Gr = 2.5 \times 10^4$ , and  $Re_A = 660$ . (b) Distribution of  $Nu_V(\Theta)$  with  $\Theta$  for  $A/D = 1.78$ ,  $Gr = 2.5 \times 10^4$ , and  $Re_A = 610$ .

cylinder. Flow visualization with the infinite fringe setting of the differential interferometer revealed that, for this case, the vibration of the cylinder caused a phenomenon of boundary-layer shedding both at the bottom and the top of the cylinder each time the cylinder changed directions. The free convective plume was shed at higher frequencies of oscillation, and a forced convective type flow was observed around the cylinder particularly at positions in the center region of oscillation. Thus, the heat-transfer rate in the region of the top of the cylinder was greatly increased over the free convective value. The symmetry of this plot about a horizontal plane through the center of the cylinder reveals the fact that the free convective influence was small for this case. The increases of the heat transfer on the sides of the cylinder reveal that the extent of the horizontal motion of the cylinder was greater than the free convective boundary-layer thickness.

In case (b) of Fig. 6, the distribution of  $Nu_V(\Theta)$  is plotted for  $A/D = 1.78$ ,  $Gr = 2.5 \times 10^4$ , and  $Re_A = 610$ . These dimensionless numbers correspond to a frequency of oscillation of  $2.3 \text{ c/s}$  and an excess temperature of  $14 \text{ deg C}$ . All values of  $Nu_V(\Theta)$  were larger than the corresponding free convective values. The plot of  $Nu_V(\Theta)$  reveals that the increases in the heat transfer for this case were primarily due to the fact that the extended horizontal motion of the cylinder was much larger than the free convective boundary-layer thickness. The free convective influence was still present near the top of the cylinder for the large amplitude, low frequency case shown in Fig. 6(b). The

dip in the plot of  $Nu_V(\Theta)$  in the region around the top of the cylinder reveals the persistence of the free convective plume and the inability of the low frequency oscillations to remove the plume from the cylinder entirely.

The plot of  $Nu_V(\Theta)$  obtained by Fand *et al.* [15] is quite different from the results shown in Fig. 6. When  $Nu_V(\Theta)$  was plotted, figure 8-shaped curves (with the pinch in the horizontal direction) were obtained, and regions (near  $\Theta = 97$  and  $263^\circ$ ) of locally reduced heat transfer were reported. The investigation reported in reference [15] utilized acoustical vibrations, and the value of  $A/D$  was approximately two orders of magnitude smaller than those reported in this study; and therefore, the results cannot be expected to compare closely with the results of this investigation.

Values for the overall average Nusselt number,  $\overline{Nu}_V$ , were calculated for all of the vibratory tests by averaging the instantaneous average Nusselt numbers over the cycle of oscillation. Comparison of  $\overline{Nu}_V$  with the average Nusselt number,  $\overline{Nu}_F$ , for the stationary cylinder at the same  $Gr$  showed that all values of  $\overline{Nu}_V$  were larger than the corresponding values of  $\overline{Nu}_F$ . Within the range of parameters tested, the largest difference between  $\overline{Nu}_V$  and  $\overline{Nu}_F$  was 200 per cent, and the smallest difference was 1 per cent.

Increases in the overall average heat-transfer rate were less than 15 per cent for  $A/D$  less than 0.24 even at average Reynolds number as high as 300. This small increase was due to the fact that the vibration of these low values of  $A/D$  did not cause the cylinder to extend beyond the thermal-boundary layer established by free convection. Therefore, there was no periodic displacement of fluid across the cylinder to cause a net increase in heat transfer. At low  $A/D$  and low vibrational intensities, natural convection dominates, and significant increases in heat transfer occur only when a critical vibrational intensity is reached and the coupling between streaming currents due to vibration and natural convective currents causes a net increase in enthalpy transport.

Due to the limitations on frequency of the vibrational equipment used in this investigation, the critical vibrational intensity could not be reached for values of  $A/D$  of 0.12 and 0.24. For large  $A/D$  the equipment was capable of exceeding the critical  $Af$  product which has been previously reported to be approximately 10 cm/s [4]. As mentioned above, the existence of the critical vibrational intensity was observed for low values of  $A/D$  because only small increases in the average heat transfer were observed at intensities below the critical value. However at  $A/D$  values above 0.24, a critical vibrational intensity did not appear to exist because the average Nusselt number increased gradually as the vibrational speed increased from zero.

When the data for  $\overline{Nu}_V$  were compared with both the forced convective curves recommended by Holman [20] and Oosthuizen and Maden [21] and the free-forced convective curves by Oosthuizen and Madan [22],  $\overline{Nu}_V$  values generally lie above both the forced and mixed convective curves except at low  $A/D$  and medium  $Re_A$  (300) where vibration had little effect on heat transfer. At higher  $Re_A$  where the oscillatory forces dominated, the value of  $\overline{Nu}_V$  appears to be parallel and above the forced convective curve. A representative plot of  $\overline{Nu}_V$  as a function of  $Re_A$  is shown in Fig. 7.

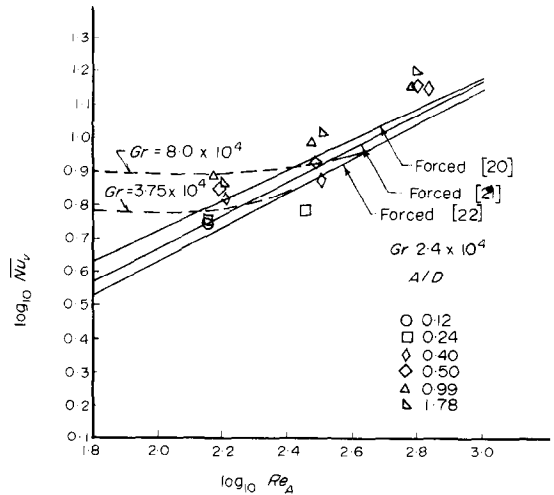


FIG. 7. Comparison of  $\overline{Nu}_V$  with forced and mixed free-forced convection for  $Gr = 2.4 \times 10^4$ .

A correlation for the overall average Nusselt number,  $\overline{Nu}_V$ , was made by using the combination of parameters discussed previously. This correlation assumes that  $\overline{Nu}_V$  is expressible as a sum of the free convective and oscillatory effects

$$\overline{Nu}_V = \overline{Nu}_F + \overline{Nu}_{osc} \tag{2}$$

or

$$\frac{\overline{Nu}_V}{\overline{Nu}_F} = 1 + \frac{\overline{Nu}_{osc}}{\overline{Nu}_F} \tag{3}$$

where  $\overline{Nu}_F$  is the contribution due to free convection, and  $\overline{Nu}_{osc}$  is the contribution due to oscillation.

All of the dimensionless parameters given in equation (1) were used to correlate the data except the Prandtl number. It was not included because the Prandtl number for atmospheric air is virtually constant for the ranges of temperatures used in this investigation. Several different forms of the correlation as well as several different groups of dimensionless numbers were used before the final correlation equation was selected. The different groups of the dimensionless parameters included:

$$Re_A, \frac{A}{D} Re_A, \frac{Re_A}{A/D}, \frac{A}{D} Gr^{1/2}, \frac{Re_A}{Gr^{1/2}}, \frac{Re_A}{(A/D)Gr^{1/2}}$$

The constants in the equation were evaluated by a least square technique. The final form of the equation

$$\overline{Nu}_V = 1 + \frac{\overline{Nu}_{osc}}{\overline{Nu}_F} = 1 + \Phi \quad (4)$$

where

$$\Phi = 0.168 \left( \frac{A}{D} \right) \left[ -32.3 + 33.3 \left( \frac{A}{D} Re_A \right)^{-0.01} \right] \times \left( \frac{A}{D} Re_A \right)^{1.09} Gr^{-0.452} \quad (5)$$

results in an average difference between the correlation and the experimental data of 6 per cent and a maximum difference for any one data point of 14 per cent. The plot of the correlation is shown in Fig. 8.

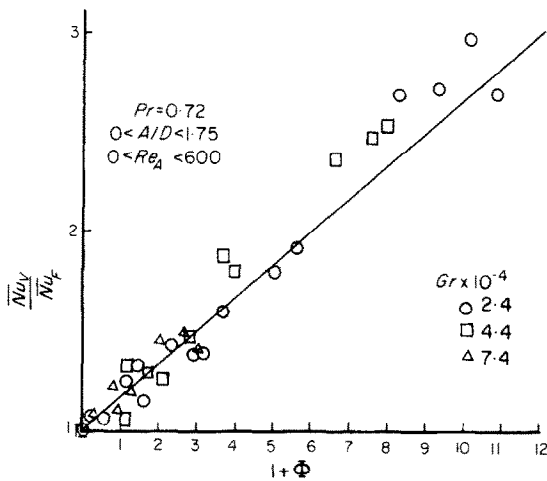


FIG. 8. Correlation for  $\overline{Nu}_V/\overline{Nu}_F$  expressed in equations (4) and (5).

The symbol  $\Phi$  in the correlation may be thought of as representing an augmentation factor whose magnitude provides a measure of the effectiveness of vibration on the heat transfer from the cylinder. Values for the augmentation factor for all vibratory tests are shown in Table 1.

The correlation of equation (4) was extended beyond the ranges of nondimensional parameters for which it was derived to see how accurately it could fit the results of Fand and Peebles [4]. In this reference, the heat-transfer results of investigations involving mechanical and acoustical vibration were compared. For the mechanical vibrational results, the values predicted by the correlation were in good agreement with the experimental values except for two points where  $Re_A$  was above 1500. The maximum value of  $Re_A$  for the tests from which the correlation was derived was 660. If the points for  $Re_A$  greater than 1500 are excluded, the average difference between the correlative value and experimental value of  $\overline{Nu}_V/\overline{Nu}_F$  was 8 per cent.

Table 1. Augmentation factor for vibrational tests  $\Phi \times 100\%$

$\frac{A}{D}$	$Af(\text{cm/s})$	$Re_A$	$Gr \times 10^{-4}$		
			2.5	4.5	7.5
0	Stationary				
1/8	2.54	150	4	3	1
1/4	2.54	150	6	8	8
	5.08	300	14	8	10
3/8	2.54	150	24		
	5.08	300	39		
	10.16	600	170		
1/2	2.54	150	33	33	25
	5.08	300	55	48	46
	10.16	600	170	140	
1	2.54	150	43	29	19
	5.08	300	79	88	49
	10.16	600	170	160	
7/4	2.54	150	38	26	15
	5.08	300	92	81	42
	10.16	600	200	150	

The correlation given in equations (4) and (5) did not adequately predict  $\overline{Nu}_V/\overline{Nu}_F$  for the acoustical vibration data in [4]. This is probably due to the fact that  $A/D$  for these data are approximately two orders of magnitude smaller than the  $A/D$  for the tests for which the correlation was derived. Fand and Peebles have stated that their results indicated that the parameter  $A/D$  was not important for the range of data in their investigation; however, for the range of parameters used in this investigation,  $A/D$  has greatly influenced the value of  $\overline{Nu}_V/\overline{Nu}_F$ .

An attempt was also made to extend the correlation to the range of parameters used by Lemlich [5]. This correlation did not successfully predict the heat-transfer rate from the wire probably due to the fact that values for  $Re_A$  and  $Gr$  were smaller by approximately one and four orders of magnitude, respectively, than the tests for which equations (4) and (5) were derived.

## CONCLUSIONS

The heat-transfer measurements have shown that vibration can significantly affect the heat-transfer rate from an isothermal, horizontal cylinder. The overall heat-transfer rates show moderate increases as the vibrational speed is increased from zero; however, instantaneous average heat-transfer rates can show significant changes with only small increases in vibrational speed. For example, increases up to 1400 per cent in the instantaneous local Nusselt number and increases in the average Nusselt number up to 200 per cent above the free convective values were measured.

For the range of dimensionless parameters investigated, the effect of vibration on the heat-transfer rate



varies significantly with the following parameters: the Grashof number, the ratio of amplitude of oscillation to cylinder diameter, and the Reynolds number based on the average cylinder speed. A correlation of the overall average Nusselt number was achieved as a function of these parameters which could accurately predict the heat transfer from a vibrating cylinder to air. An extension of the correlation to the mechanical vibration data of [4] which involves higher Reynolds numbers has indicated that the correlation is also within 8 per cent of data having an average Reynolds number as high as 1500. However, attempts to extend the correlation to vibrations involving amplitudes to diameter ratios frequently produced in acoustical vibrational studies was not successful.

The ratio of amplitude of vibration to cylinder diameter has shown to exhibit a strong influence on the heat-transfer rate from a vibrating cylinder. A critical value of vibrational intensity was observed for values of  $A/D$  of  $1/8$  and  $1/4$ ; however, for larger values of  $A/D$ , a critical intensity was not observed, and the heat-transfer rate was found to increase as soon as the velocity of the cylinder was increased from zero.

The heat-transfer rates for tests conducted at higher Grashof numbers were less affected by vibration because the free convective characteristics of the flow pattern were more significant. The main effect of increasing the Reynolds number was a reduction of the free convective influence on the heat-transfer mechanism with a resulting increase in importance of the forced convective and the oscillatory mechanisms.

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### MESURE INTERFEROMETRIQUE DU TRANSFERT THERMIQUE LOCAL ET INSTANTANE PAR UN CYLINDRE HORIZONTAL, ISOTHERME ET VIBRANT

**Résumé**—Les coefficients instantanés et locaux de transfert thermique sont mesurés pour la convection libre autour d'un cylindre isotherme qui vibre sinusoidalement dans un plan horizontal. Les résultats montrent que le coefficient de transfert est fortement dépendant du rapport de l'amplitude de la vibration au diamètre du cylindre,  $A/D$ . Pour des valeurs de  $A/D$  inférieures à  $1/4$  environ, on observe une intensité de vibration critique au dessous de laquelle la vibration cause un faible accroissement du flux thermique: Pour  $A/D$  supérieur à  $1/4$ , l'intensité critique de la vibration ne semble pas exister car le transfert thermique augmente dès que la vitesse de vibration croît depuis zéro. On présente les résultats sous une forme unifiée qui précise l'accroissement du transfert moyen entre le cylindre et l'air en convection naturelle. Ces résultats s'appliquent aussi au cas des intensités de vibration plus élevées.

### INTERFEROMETRISCHE MESSUNG DES LOKALEN MOMENTANEN WÄRMEÜBERGANGS AN EINEM HORIZONTAL EN VIBRIERENDEN ISOTHERMEN ZYLINDER

**Zusammenfassung**—Bei freier Konvektion wurden an einem isothermen, in der Horizontalen sinusförmig schwingenden Zylinder lokale momentante Wärmeübergangskoeffizienten gemessen. Die Ergebnisse zeigen, daß der Wärmeübergangskoeffizient stark vom Verhältnis der Amplitude zum Zylinderdurchmesser  $A/D$  abhängt. Bei Werten von  $A/D$  näherungsweise kleiner als  $1/4$  wurde eine kritische Vibrationsintensität beobachtet, unterhalb der die Vibration den Wärmeübergang nur geringfügig vergesserte; bei  $A/D = 1/4$  schien die kritische Vibrationsintensität nicht zu bestehen, da der Wärmeübergang vom Zylinder sofort anstieg, wenn die Vibrationsgeschwindigkeit von Null anstieg. Es wird eine Korrelation der Werte angegeben, die den Anstieg des mittleren Wärmeübergangs vom Zylinder an Luft über den Wert der freien Konvektion hinaus wiedergibt. Diese Korrelation gilt auch für Werte bei viel größeren Vibrationsintensitäten.

### ИНТЕРФЕРОМЕТРИЧЕСКОЕ ИЗМЕРЕНИЕ МГНОВЕННЫХ ЛОКАЛЬНЫХ КОЭФФИЦИЕНТОВ ПЕРЕНОСА ТЕПЛА ОТ ИЗОТЕРМИЧЕСКОГО ЦИЛИНДРА, КОЛЕБЛЮЩЕГОСЯ В ГОРИЗОНТАЛЬНОЙ ПЛОСКОСТИ

**Аннотация** — Измерялись локальные мгновенные коэффициенты теплоотдачи при свободной конвекции от изотермического цилиндра, колеблющегося синусоидально в горизонтальной плоскости. Результаты показывают, что коэффициент теплоотдачи находится в сильной зависимости от отношения амплитуды колебания к диаметру цилиндра,  $A/D$ . Для значений  $A/D$  меньше чем, примерно,  $1/4$  наблюдался критический уровень вибрации, ниже которого колебания вызывали только незначительное увеличение интенсивности теплоотдачи; однако, при значениях  $A/D$  больших, чем  $1/4$ , критического уровня колебаний не наблюдалось, так как теплоотдача от цилиндра увеличивалась от нулевого значения по мере возрастания интенсивности вибрации. Получено соотношение, позволяющее определить увеличение среднего значения теплопереноса от цилиндра по сравнению с переносом при «чистой» свободной конвекции. Это соотношение может быть также применено для значительно больших интенсивностей вибрации.